Interaction of pulses in the nonlinear Schrödinger model

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(Received 23 December 2002; published 14 May 2003)

The interaction of two rectangular pulses in the nonlinear Schrödinger model is studied by solving the appropriate Zakharov-Shabat system. It is shown that two real pulses may result in an appearance of moving solitons. Different limiting cases, such as a single pulse with a phase jump, a single chirped pulse, in-phase and out-of-phase pulses, and pulses with frequency separation, are analyzed. The thresholds of creation of new solitons and multisoliton states are found.

DOI: 10.1103/PhysRevE.67.056610 PACS number(s): 42.65.Tg, 42.65.Jx

I. INTRODUCTION

The nonlinear Schrödinger (NLS) equation is an important model of the theory of modulational waves. It describes the propagation of pulses in optical fibers [1,2], the dynamics of laser beams in a Kerr media, or the nonlinear difraction [3], waves in plasma [4], and the evolution of a Bose-Einstein condensate wave function [5]. The NLS equation is written in dimensionless form as

$$iu_z + u_{xx}/2 + |u|^2 u = 0, \tag{1}$$

where $u(x,z)$ is a slowly varying wave envelope, z is the evolutional variable, and x is associated with the spatial variable.

An exact solution of the NLS equation has a form of a soliton:

$$u(x,z) = 2\eta\,\mathrm{sech}[2\eta(x+2\xi z - x_0)] \times \exp[-2i\xi x - 2i(\xi^2-\eta^2)z + i\phi_0], \tag{2}$$

where 2η and 2ξ are amplitude, or the inverse width, and the velocity of the soliton, x_0 and ϕ_0 are the initial position and phase, respectively. The soliton represents a basic mode and plays a fundamental role in nonlinear processes. The dynamics of NLS solitons and single pulses even in the presence of various perturbations is well understood (see, e.g., Refs. [1,2], and references therein). However, the evolution of several pulses is not studied in detail. In recent works [6] (see also Ref. [2]) mostly an interaction of solitons and near-soliton pulses was considered. A study of *near-soliton* pulses, especially a use of the effective particle approach, often results in small variation of soliton parameters, including the soliton velocities and as a consequence weak repulsion or attraction of solitons. Such a study does not involve a possibility of an appearance of additional solitons. However, for many applications it is necessary to consider the interaction of pulses with arbitrary amplitudes or pulses with different parameters. For example, in optical communication systems with the wavelength division multiplexing (WDM), the initial signal consists of several solitons with different frequencies. An estimation of the critical separation between pulses is important for determination of the repetition rate of a particular transmission scheme.

In the present work, the interaction of two pulses in the NLS model is studied both theoretically and numerically. We show the presence of different scenarios of the behavior, depending on the initial parameters of the pulses, such as the pulse areas, the relative phase shift, the spatial and frequency separations. One of our main observation is a fact that a pure real initial condition of the NLS equation can result in additional *moving* solitons. As a consequence the number of solitons, emerging from two pulses separated by some distance, can be larger than the sum of the numbers of solitons, emerging from each pulse. Such properties were also found for the Manakov system [7], which is a vector generalization of the NLS equation. The scalar NLS equation was studied in Ref. [7] as a particular case. Later similar results and approximation formulas for the soliton parameters were obtained in papers [8,9] (see also Ref. [10]). In works [7–10] mostly the interaction of *real* pulses was analyzed, while here we consider pulses with a nonzero relative phase shift and frequency separation. A preliminary version of this study was presented in work [11].

The paper is organized as following. The linear scattering problem associated with the NLS equation is considered in Sec. II. We also present the general solution of the problem for the case of two rectangular pulses. In Sec. III, we study different particular cases, such as two in-phase pulses, two out-of-phase pulses, a single pulse with a phase jump, a single chirped pulse, and two pulses with the frequency separation. The results and conclusions are summarized in Sec. IV.

II. DIRECT SCATTERING PROBLEM

In this paper we are interested only in an asymptotic state of the pulse interaction. In order to simplify the problem and to obtain exact results we consider the interaction of two *rectangular* pulses (“boxes”). Therefore we take the following initial conditions for Eq. (1):

$$u(x,0) \equiv U(x) = \begin{cases} Q_1 \exp[2i\nu_1 x] & \text{for} \quad x_1 < x < x_2 \\ Q_2 \exp[2i\nu_2 x] & \text{for} \quad x_3 < x < x_4 \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

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where Q_1 and Q_2 are the complex constant amplitudes, $w_1 \equiv x_2 - x_1$ and $w_2 \equiv x_4 - x_3$ are the pulse widths, and $2\nu_1$ and $2\nu_2$ are the detunings.

It is known that the NLS equation is integrable by the inverse scattering transform method [3]. As follows from this fact, initial conditions, which decrease sufficiently fast at $x = \pm\infty$, result in a set of solitons and linear waves (so called, radiation). The number N and parameters of solitons emerging from an initial condition are found from the solution of the Zakharov-Shabat scattering problem [3]:

$$i\frac{\partial \psi_1}{\partial x} - iU(x)\psi_2 = \lambda\psi_1,$$

$$-i\frac{\partial \psi_2}{\partial x} - iU^*(x)\psi_1 = \lambda\psi_2, \qquad (4)$$

with the following boundary conditions:

$$\Psi_{x\to-\infty} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\lambda x}, \quad \Psi_{x\to\infty} = \begin{pmatrix} a(\lambda)e^{-i\lambda x} \\ b(\lambda)e^{i\lambda x} \end{pmatrix}. \qquad (5)$$

Here $\Psi(x)$ is an eigenvector, λ is an eigenvalue, $a(\lambda)$ and $b(\lambda)$ are the scattering coefficients, and an asterisk means a complex conjugate. The number N is equal to the number of poles $\lambda_n \equiv \xi_n + i\eta_n$, where $n = 1, \ldots, N$, and $\eta_n > 0$, of the transmission coefficient $1/a(\lambda)$. Each λ_n is invariant on z. If all ξ_n are different then $u(x,z)$ at $z\to\infty$ represents a set of solitons, each in the form of Eq. (2) with $\eta = \eta_n$ and $\xi = \xi_n$. If real parts of several λ_n are equal then a formation of a neutrally stable bound state of solitons is possible.

The solution of the Zakharov-Shabat problem (4) with potential (3) is written as

$$\begin{aligned} a(\lambda) = {} & e^{i(\lambda+\nu_1)w_1} e^{i(\lambda+\nu_2)w_2} \\ & \times \Bigg\{ \left[\cos(k_1 w_1) - i\frac{(\lambda+\nu_1)}{k_1}\sin(k_1 w_1) \right] \\ & \times \left[\cos(k_2 w_2) - i\frac{(\lambda+\nu_2)}{k_2}\sin(k_2 w_2) \right] \\ & - \frac{Q_1^* Q_2}{k_1 k_2} e^{-2i(\lambda+\nu_1)x_2} e^{2i(\lambda+\nu_2)x_3} \\ & \times \sin(k_1 w_1)\sin(k_2 w_2) \Bigg\}, \end{aligned} \qquad (6)$$

$$\begin{aligned} b(\lambda) = {} & e^{i(\lambda+\nu_1)w_1} e^{-i(\lambda+\nu_2)w_2} \Bigg\{ -\frac{Q_1^*}{k_1} e^{-2i(\lambda+\nu_1)x_2}\sin(k_1 w_1) \\ & \times \left[\cos(k_2 w_2) + i\frac{(\lambda+\nu_2)}{k_2}\sin(k_2 w_2) \right] \\ & - \frac{Q_2^*}{k_2} e^{-2i(\lambda+\nu_2)x_3}\sin(k_2 w_2) \\ & \times \left[\cos(k_1 w_1) - i\frac{(\lambda+\nu_1)}{k_1}\sin(k_1 w_1) \right] \Bigg\}, \end{aligned} \qquad (7)$$

where $k_j = [(\lambda+\nu_j)^2 + |Q_j|^2]^{1/2}$.

Since the linear operator in Eq. (4) is not Hermitian, complex eigenvalues are possible even for real $u(x,0)$ (e.g., see Sec. III A). Though this is an obvious fact, "an interesting 'folklore' property seems to have arisen in the literature over the last 25 years, namely, that only pure imaginary EVs (eigenvalues) can occur for symmetric real valued potentials" [8]. As demonstrated below, the statement [Theorem (III) in Sec. II] of paper [12], which claims this result, is incorrect. An existence of eigenvalues with nonzero real parts for Zakharov-Shabat problem with pure real potential was first shown in paper [7].

Equations (6) and (7) represent a general solution of the scattering problems (4) and (5) with initial condition (3). Applications of these equations to particular cases of the pulse interaction are considered in the following section.

III. RESULTS

A. Interaction of in-phase pulses with equal amplitudes

1. Properties of eigenvalues

Here we analyze a simple case of two real pulses, separated by a distance $L \equiv x_3 - x_2$, with zero detuning, i.e., $Q_1 = Q_2 = Q_0$, $w_1 = w_2 \equiv w$, and $\nu_1 = \nu_2 = 0$, where Q_0 is real. Then using Eq. (6), the equation for discrete spectrum is written as

$$F(\lambda, Q_0, w) \pm \frac{Q_0}{k} e^{i\lambda L}\sin(kw) = 0, \qquad (8)$$

where $F(\lambda,Q,w) \equiv \cos(kw) - i\lambda\sin(kw)/k$, and $k = (\lambda^2 + Q^2)^{1/2}$. Note that $F(\lambda,Q_0,w) = 0$ determines the discrete spectrum for a single box with zero detuning [13]. Therefore the second term in Eq. (8) can be associated with the result of nonlinear interference. Recall also that for a single box with amplitude Q_0 and width w, the number N_{SB} of emerging solitons is determined as [3] $N_{SB} = \mathrm{int}(Q_0 w/\pi + 1/2)$, where int() means an integer part. Results for the two boxes are reduced to those for a single box in limiting cases $L = 0$ and $L = \infty$.

As shown by Klaus and Shaw [8], the Zakharov-Shabat problem with a "single-hump" real initial condition admits pure imaginary eigenvalues only, i.e., solitons with zero velocity. We show that the case of two pulses provides much richer dynamics.

Let us now compare the properties of eigenvalues at different $S \equiv Q_0 w$ (Fig. 1). In Fig. 1, as well as in subsequent figures of the paper, all variables are dimensionless. In the first two cases, $S = 1.8$ and $S = 2.0$, there is one soliton at $L = 0$ and there are two solitons at $L = \infty$, while in the case $S = 2.5$ there are two solitons in both limits. The dependence of eigenvalues on L at $S = 2.5$ is obvious, while that at $S = 1.8$ and 2.0 looks unexpected. First, the number of solitons at intermediate L is larger than that in the limits $L = 0$ and $L = \infty$. Second, the two real boxes lead to eigenvalues with nonzero real part. Third, for $S = 2.0$ there is a "fork" bifurcation at $L = L_F \approx 4.1$, when two eigenvalues coincide. At larger L three pure imaginary eigenvalues constitute a three-soliton state, so that the limiting two-soliton case at $L \to \infty$ is

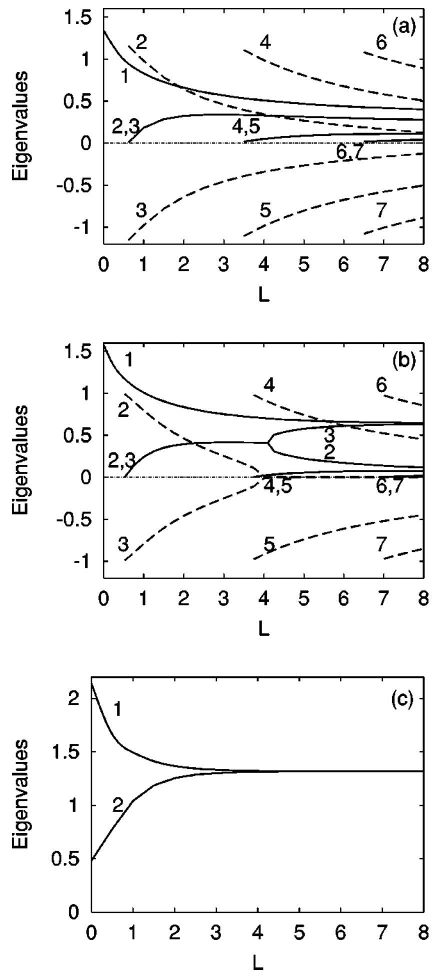

FIG. 1. In-phase pulses: the dependence of real (dashed lines) and imaginary (solid lines) parts of λ_n on the separation distance, $w=1$. The numbers near the lines correspond to n. (a) $Q_0=1.8$, (b) $Q_0=2.0$, (c) $Q_0=2.5$.

realized as a limit of a three-soliton solution with an amplitude of the third soliton tending to zero.

Results of numerical simulations of the NLS equation (1) agree with the analysis of Eq. (8). For example, as shown in Fig. 2, in accordance with Fig. 1(b) there are one fixed and two moving solitons at $S=2.0$ and $L=2$, and there are a three-soliton state and two moving solitons at $S=2.0$ and $L=5$. Note that an appearance of moving solitons and multisoliton states is not related to the rectangular form of initial pulses. For example, an initial condition $u(x,0)=0.7[\mathrm{sech}(x+2.5)+\mathrm{sech}(x-2.5)]$ also results in moving solitons.

Below we discuss in details the behavior of the eigenvalues, namely, we find a threshold of appearance of new roots, estimate a number of emerging solitons, and calculate a threshold for the fork bifurcation. It should be mentioned that eigenvalues with a nonzero real part do not exist only at $S=[3\pi/4,3.3]$ and $S=[7\pi/4,5.51]$ (see Sec. III A 2), so that the dependence at $S=2.5$ is rather an exception than a general rule. This result allows to understand why moving solitons are not observed in interaction of near-soliton pulses with an area $S\approx\pi$.

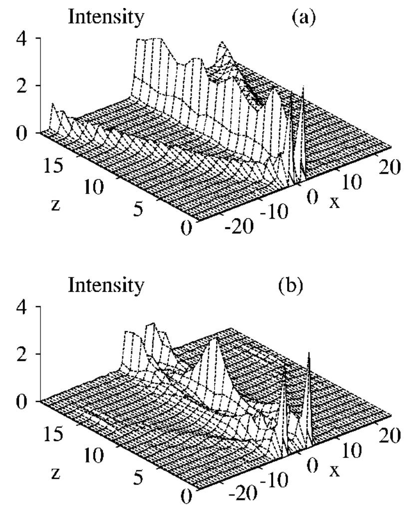

FIG. 2. Evolution of two rectangular pulses, $Q_0=2$, $w=1$. (a) One fixed soliton and two moving solitons at $L=2$. (b) Three-soliton state at $L=5$.

2. Appearance of new eigenvalues

Solving numerically Eq. (8), one can conclude that new eigenvalues penetrate to the upper half plane of λ in pairs by crossing the real axis. Therefore, the bifurcation parameter can be found from Eq. (8), assuming that $\lambda=\beta$, where β is real:

$$\cot y=\pm\frac{\sqrt{2S^2-y^2}}{y}, \tag{9}$$

$$\beta=\pm Q_0\sin(\beta L). \tag{10}$$

Here $y=\kappa w$, $\kappa=(\beta^2+Q_0^2)^{1/2}$, and the signs are taken such that $\tan(y)\tan(\beta L)<0$ is satisfied. As follows from the definition of y and Eq. (9), one has $S\leqslant y<2S$.

Analysis of Eqs. (9) and (10) results in the following conclusions.

(i) As follows from Eq. (9), the number N_{PP} of the penetration points depends only on S and is determined from

$$N_{PP}=4(m-n+1)-2\theta\left[S-\left(n+\frac{1}{4}\right)\pi\right]-2\theta\left[S-\left(n+\frac{3}{4}\right)\pi\right]-4\theta(S_m-S)\quad\text{for } S\geqslant 3\pi/4, \tag{11}$$

where $m=\mathrm{int}(\sqrt{2}\,S/\pi)$, $n=\mathrm{int}(S/\pi)$, $\theta(x)$ is the Heaviside function, and S_m is a root of

$$\tan(\sqrt{2S_m^2-1})=\sqrt{2S_m^2-1}, \tag{12}$$

which satisfies $m\pi\leqslant(2S_m^2-1)^{1/2}<(m+1)\pi$. It is easy to find that $N_{PP}=0$ for $S<\pi/4$ and $N_{PP}=2$ for $\pi/4<S<3\pi/4$. Equation (12) defines such values of $S=S_m$, when the right-hand side of Eq. (9) with plus sign touches $\cot y$ curve. All penetration points β_j, where $j=1,\ldots,N_{PP}$, are symmetrically situated with respect to $\beta=0$.

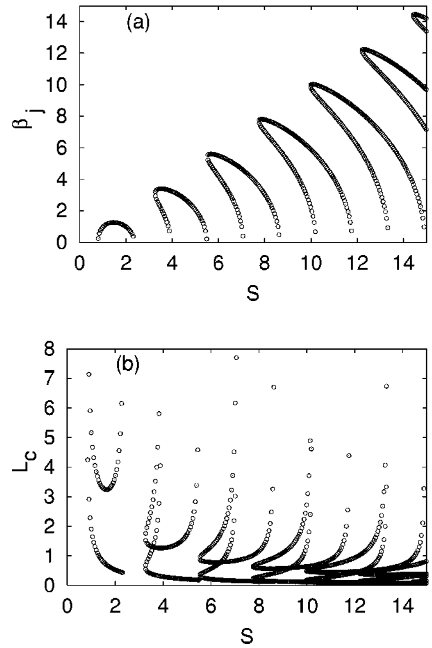

FIG. 3. (a) The dependence of β_j on S. (b) Threshold L_C, when eigenvalues cross the real axis of the λ plane, as a function of S.

(ii) All roots $|\beta_j| \leqslant Q_0$, which follows from $2S^2 - y^2 \geqslant 0$.

(iii) For every β_j, Eq. (10) defines the separation distance $L = L_C$, when eigenvalues cross the real axis.

(iv) As follows from Eq. (10) there is an *infinite* number of thresholds L_C for a given β_j. However, the total number of eigenvalues in the upper half plane of λ is, most probably, finite, because for some L_C eigenvalues pass to the upper half plane, and for other L_C eigenvalues go to the lower half plane. The direction of eigenvalue motion is defined by the derivative $d\lambda/dL$ at $\lambda = \beta_j$.

The position of penetration points β_j as a function of S is shown in Fig. 3(a), where only positive β_j are presented. As follows from Eq. (9) the number N_{PP} decreases by 2, when S passes $(2l+1)\pi/4$, where $l = 1,2 \ldots$, and N_{PP} increases by 4, when S exceeds S_m [see Eq. (12)]. Therefore one can obtain that Eq. (9) has no roots only at $S = [3\pi/4, S_2]$ and at $S = [7\pi/4, S_3]$, where $S_2 \approx 3.26$ and $S_3 \approx 5.51$ are found from Eq. (12). This property is clearly seen in Fig. 3. The dependence of L_C on S is presented in Fig. 3(b). Only the thresholds, such that $\beta_j L_C = [0, 2\pi]$, are shown for each β_j.

3. Thresholds of the fork bifurcation

Here we analyze a bifurcation, when a pair of complex eigenvalues becomes pure imaginary, e.g., $L_F \approx 4.1$ in Fig. 1(b). The equation that determines pure imaginary eigenvalues can be obtained from Eq. (6) with $\mathrm{Re}[\lambda] = 0$, i.e., $\lambda = i\gamma$:

$$\cot y = \frac{-\sqrt{S^2 - y^2} \pm S \exp[-\sqrt{S^2 - y^2} L/w]}{y}. \qquad (13)$$

Here $y = \kappa w$, $\kappa = (-\gamma^2 + Q_0^2)^{1/2}$. It is easy to show that κ^2 should be positive (there is no real solution for $\kappa^2 < 0$). As a consequence, all pure imaginary eigenvalues satisfy $\gamma_j \leqslant Q_0$.

The value of $L = L_F$, when new pure imaginary root of Eq. (13) appears, corresponds to the fork bifurcation. The bifurcation threshold L_F can be found from the condition that the functions, corresponding to the right-hand side of Eq. (13), touch the $\cot y$ curve.

Lower bound of the number of the pure imaginary eigenvalues is found as $N_{LB} = \mathrm{int}(2S/\pi + 1/2)$. This number is an actual number of the pure imaginary eigenvalues for all S, except of the regions where

$$\frac{\pi}{2} + \pi l < S < \frac{3\pi}{4} + \pi l, \quad l = 0,1, \ldots . \qquad (14)$$

If S satisfies Eq. (14) then the number of pure imaginary eigenvalues can be either N_{LB} or $N_{LB}+2$, depending on whether $L < L_F(S)$ or $L > L_F(S)$, respectively. Therefore the appearance of new pure imaginary eigenvalues, in other words, the fork bifurcation, is possible only if S satisfies Eq. (14). Figure 4 represents the dependence of L_F on S, where only one interval, corresponding to $l = 1$ in Eq. (14), is shown; the behavior for $l > 1$ is similar. One can calculate that $L_F(S = 1.8) = 10.4$, which is why the fork bifurcation is not seen in Fig. 1(a).

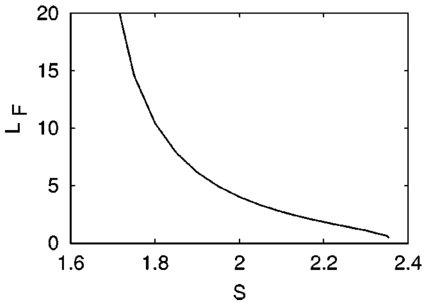

FIG. 4. Threshold L_F of the fork bifurcation as a function of S.

B. Two out-of-phase pulses with equal amplitudes

In this section we study the influence of constant phase shift on the pulse interaction, i.e., we consider $Q_1 = Q_0 \exp(-i\alpha)$, $Q_2 = Q_0 \exp(i\alpha)$, where Q_0 and α are real, $w_1 = w_2 \equiv w$, and $\nu_1 = \nu_2 = 0$. The nonzero relative phase shift 2α changes greatly the properties of the eigenvalues, so that the behavior presented in Sec. III A is hard to realize in experiments, because it is difficult to prepare two pulses exactly in phase. The phase shift breaks the *simultaneous* appearance of a pair of solitons at $\lambda = \pm\beta_j$, and affects the fork bifurcation.

For $\alpha \neq 0$, the equations for eigenvalues and for penetration points can be obtained from Eq. (8) and Eqs. (9) and (10) by changing $\lambda L \to \lambda L + \alpha$ and $\beta L \to \beta L + \alpha$ in the exponent and sinus functions, respectively. Therefore the number and positions of the penetration points are the same as for the case $\alpha = 0$. As for the threshold L_C, it is shifted on the value α/β_j, so that $L_C(\alpha) = L_C(\alpha = 0) + \alpha/\beta_j$, where only $L_C \geqslant 0$ should be taken into account.

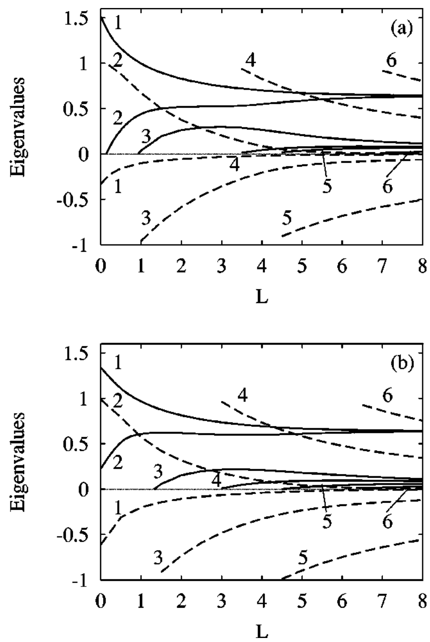

FIG. 5. Out-of-phase pulses: The dependence of real (dashed lines) and imaginary (solid lines) parts of λ_n on L for $Q_0=2$, $w=1$. The numbers near the lines correspond to n. (a) $\alpha=\pi/8$, (b) $\alpha=\pi/4$.

The influence of the phase shift on the distribution of eigenvalues is shown in Fig. 5. As seen, now new eigenvalues appear one by one, not in pairs, and, as a consequence, the fork bifurcation disappears. Further, the real parts of the roots do not vanish at finite L, but decrease smoothly. This means that the presence of the phase shift breaks up a multisoliton state, which is known to be neutrally stable to perturbations.

At $L=0$, the phase shift corresponds to the phase jump of a single pulse. Such a phase jump can result in an appearance of additional solitons as shown in Fig. 5(b). The threshold of the phase shift α_{th}, when the first new soliton appears, can be found from the condition $\alpha_{\rm th}=|\beta_1|L_C(\alpha=0)$, where β_1 is the position of the penetration point nearest to zero.

C. Two pulses with frequency separation

In this section we analyze initial condition (3) with the following parameters $Q_1=Q_2=Q_0$, $w_1=w_2=w$, $-\nu_1=\nu_2=\nu$, where Q_0 is a real constant. This case models the wavelength division multiplexing in optical fibers, the case when an input signal consists of two or more pulses with different frequencies. Actually, since Q_0 can be taken sufficiently large we consider the interaction of multisoliton states. The detailed analysis of the interaction of sech pulses at different frequencies is presented in papers [14] and in review [15]. In particular, the authors of papers [14,15] consider the evolution of a superposition of N solitons with the same position of the centers, but with different frequencies. As shown in these works there is a critical frequency separation, above which N solitons with almost equal amplitudes emerge. Below this critical value the number of emerging solitons can not equal N and their amplitudes can appreciably differ from each other. It was also demonstrated that an introduction of a time shift between pulses results in a decrease of the frequency separation threshold. The geometry described by Eq. (3) corresponds to the combination of the WDM and time-division multiplexing schemes, therefore our study can give some insight into such a behavior of the threshold. Moreover, the authors of works [14,15] mostly used the perturbation technique and numerical simulations, while in the present paper we deal with an exact solution of the Zakharov-Shabat problem.

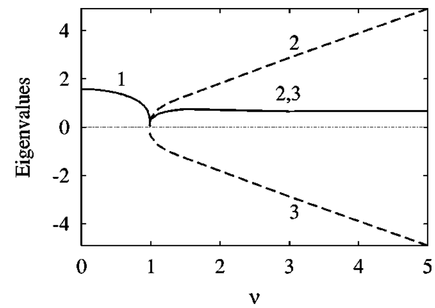

FIG. 6. Single chirped pulse; the dependence of real (dashed lines) and imaginary (solid lines) parts of λ_n on ν for $Q_0=2$, $w=1$, $L=0$. The numbers near the lines correspond to n.

First let us consider the case $L=0$ that corresponds to the case of a single *chirped* pulse of width $2w$. The dependence of the eigenvalues on ν, which plays here the role of a chirp parameter, is shown in Fig. 6. At small ν the interaction of the pulse components is strong, so that there is one pure imaginary eigenvalue, or a single soliton with zero velocity. At larger ν the frequency difference of the pulse components results in a repulsion of the components, or a pulse splitting.

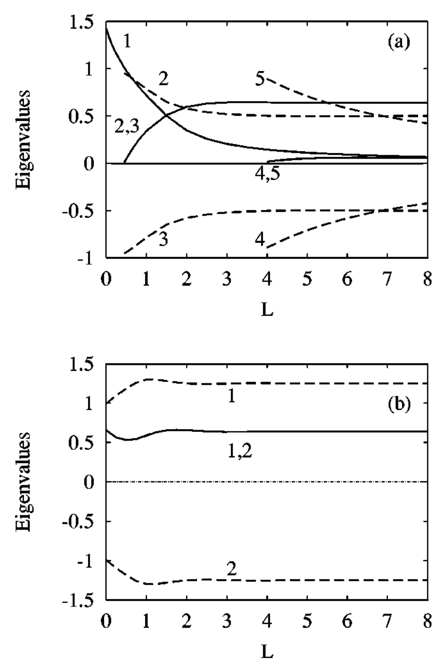

FIG. 7. Pulses with frequency separation: The dependence of real (dashed lines) and imaginary parts (solid lines) of λ_n on L for $Q_0=2$, $w=1$. The numbers near the lines correspond to n. (a) $\nu=0.5$, (b) $\nu=1.25$.

At sufficiently large ν the velocities of emerging solitons tend, as expected, to $\pm 2\nu$.

There is also a narrow region of ν, e.g., $\nu=[0.98,0.99]$ in Fig. 6, where three solitons, one fixed and two moving solitons, exist. This region separates two different types of the evolution of a chirped pulse. The left boundary, which corresponds to the appearance of new eigenvalues, of the region is found from the condition similar to that considered in Sec. III A 2. The right boundary is found from $a(\lambda=0)=0$, which defines the values of ν as a function of the other parameters, when the pure imaginary root disappears.

The dependence of the spectrum on L is presented in Fig. 7. At small ν [Fig. 7(a)] we see again an appearance of additional solitons similar to the case $\nu=0$ (Fig. 1). At larger ν [Fig. 7(b)] the repulsion is so strong that it suppresses the appearance of small-amplitude solitons. Therefore there is a threshold of the frequency separation above which the interaction of two pulses is negligible. This result is in agreement with the conclusions of paper [15].

IV. CONCLUSION

The interaction of two pulses in the NLS model is studied by means of the solution of the associated scattering problem. The strong dependence of the dynamics on the parameters of the initial pulses is shown. For intermediate separation distances L the existence of additional moving solitons is possible even in the case of two in-phase pulses with the same frequencies. These additional solitons can be considered as a result of the nonlinear interference of pulses. The phase shift of two pulses removes a degeneracy in the behavior, namely it affects the symmetry of the parameters of emerging solitons and results in a breakup of multisoliton states peculiar to the in-phase case. It is also shown that the strong frequency separation suppresses the appearance of additional solitons. The results obtained in the present paper can be useful for the analysis of the transmission capacity of communication systems and for interpretation of experiments on the interaction of two laser beams in nonlinear media. Recently, the generation of up to ten solitons has been observed experimentally in quasi-one-dimensional Bose-Einstein condensate of ^{7}Li with attractive interaction [16]. Our study can be also helpful for interpretation of this experiment.

ACKNOWLEDGMENTS

This research was partially supported by the Foundation for Support of Fundamental Studies, Uzbekistan (Grant No. 15-02) and by FAPESP (Brazil).

[1] G.P. Agrawal, *Nonlinear Optics* (Academic, San Diego, 1989); F. Kh. Abdullaev, S.A. Darmanyan, and P.K. Khabibullaev, *Optical Solitons* (Springer-Verlag, Heidelberg, 1993).

[2] A. Hasegawa and Yu. Kodama, *Solitons in Optical Communications* (Clarendon, Oxford, 1995).

[3] V.E. Zakharov and A.B. Shabat, Zh. Eksp. Teor. Fiz. **61**, 118 (1971) [Sov. Phys. JETP **34**, 62 (1972)].

[4] See, e.g., H. Ikezi, in *Solitons in Action*, edited by K. Longren and A. Scott (Academic, New York, 1978).

[5] E.P. Gross, Nuovo Cimento **20**, 454 (1961); J. Math. Phys. **4**, 195 (1963); L.P. Pitaevskii, Zh. Eksp. Teor. Fiz. **40**, 646 (1961) [Sov. Phys. JETP **13**, 451 (1961)].

[6] C. Desem and P.L. Chu, in *Optical Solitons-Theory and Experiment*, edited by J.R. Taylor (Cambridge University Press, Cambridge, 1992), Chap. 5; D. Anderson and M. Lisak, Opt. Lett. **18**, 790 (1986); V.I. Karpman and V.V. Solov'ev, Physica D **3**, 487 (1981).

[7] F.Kh. Abdullaev and E.N. Tsoy, Physica D **161**, 67 (2002).

[8] M. Klaus and J.K. Shaw, Phys. Rev. E **65**, 036607 (2002).

[9] M. Desaix, D. Anderson, L. Helczynski, and M. Lisak, in *Nonlinear Guided Waves and Their Applications*, Streza, Italy, Technical Digest of the OSA International Workshop Vol. 80 (Optical Society of America, Washington, DC, 2002).

[10] Recently we became aware of the work by M. Desaix, D. Anderson, L. Helczynski, and M. Lisak, Phys. Rev. Lett. **90**, 013901 (2003), which deals with real initial conditions. However, here we consider more general complex initial conditions [see Eq. (3)] and study different scenarios of the pulse interaction depending on space and frequency separations, phase shift, and pulse areas.

[11] E.N. Tsoy and F. Kh. Abdullaev, in *Nonlinear Guided Waves and Their Applications*, (Ref. [9]).

[12] J. Satsuma and N. Yajima, Suppl. Prog. Theor. Phys. **55**, 284 (1974).

[13] S.V. Manakov, Zh. Eksp. Teor. Fiz. **65**, 1392 (1973) [Sov. Phys. JETP **38**, 693 (1974)].

[14] P.A. Andrekson, N.A. Olson, J.R. Simpson, T. Tanbun-Ek, R.A. Logan, P.C. Becker, and K.W. Wecht, Appl. Phys. Lett. **57**, 1715 (1990); Y. Kodama and A. Hasegawa, Opt. Lett. **16**, 208 (1991); C. Etrich, N.-C. Panoiu, D. Mihalache, and F. Lederer, Phys. Rev. E **63**, 016609 (2001); N.-C. Panoiu, I.V. Mel'nikov, D. Mihalache, C. Etrich, and F. Lederer, Phys. Rev. E **60**, 4868 (1999).

[15] N.-C. Panoiu, I.V. Mel'nikov, D. Mihalache, C. Etrich, and F. Lederer, J. Opt. B: Quantum Semiclassical Opt. **4**, R53 (2002).

[16] K. Strecker, G. Partridge, A. Truscott, and R. Hulet, Nature (London) **417**, 150 (2002).